

A helyettesítéses módszer (Substitučná metóda)

Ennek az eljárásnak az alapját az összetett függvény deriválása adja.

$$(f \circ g)'(x) = \{f[g(x)]\}' = f'[g(x)] \cdot g'(x)$$

mindkét oldalt integrálva kapjuk

$$\int \{f[g(x)]\}' dx = \int f'[g(x)] \cdot g'(x) dx$$

T. $\int f(x) dx = \int f(g(t)) \cdot g'(t) dt = f[g(x)] + c$

példa:

$$\int \frac{1}{4x-2} dx = [*] \int \frac{1}{t} \frac{dt}{4} = \frac{1}{4} \int \frac{1}{t} dt = \frac{1}{4} \ln |t| + c = \frac{\ln|4x-2|}{4} + c$$

$$* \quad t = 4x - 2$$

$$\frac{dt}{dx} = 4$$

$$\frac{dt}{4} = dx$$

$$\int \sin 7x dx = [*] \int \sin t \frac{dt}{7} = \frac{-\cos t}{7} + c = -\frac{\cos 7x}{7} + c$$

$$* \quad t = 7x$$

$$\frac{dt}{dx} = 7$$

$$\frac{dt}{7} = dx$$

$$\int e^{x+1} dx = [*] \int e^t dt = e^t + c = e^{x+1} + c$$

$$* \quad t = x + 1$$

$$\frac{dt}{dx} = 1$$

$$dt = dx$$

$$\int \operatorname{tg} x dx = \int \frac{\sin x}{\cos x} dx = [*] \int \frac{\sin x}{t} \frac{dt}{-\sin x} = - \int \frac{1}{t} dt = -\ln |t| + c = -\ln |\cos x| + c$$

$$* \quad t = \cos x$$

$$\frac{dt}{dx} = -\sin x$$

$$\frac{dt}{-\sin x} = dx$$

$$\int \operatorname{cotg} x dx = \int \frac{\cos x}{\sin x} dx = [*] \int \frac{\cos x}{t} \frac{dt}{\cos x} = \int \frac{1}{t} dt = \ln |t| + c = \ln |\sin x| + c$$

$$* \quad t = \sin x$$

$$\frac{dt}{dx} = \cos x$$

$$\frac{dt}{\cos x} = dx$$

$$\int \frac{x-1}{x+2} dx = \int \frac{x+2-3}{x+2} dx = \int \frac{x+2}{x+2} - \frac{3}{x+2} dx = \int 1 - \frac{3}{x+2} dx = \int 1 dx - \int \frac{3}{x+2} dx =$$

$$= [*] x - \int \frac{3}{t} dt = x - 3 \ln |t| + c = x - 3 \ln |x + 2| + c$$

$$* \quad t = x + 2$$

$$\frac{dt}{dx} = 1$$

$$dt = dx$$

$$\int \frac{4x+3}{x-5} dx = \int \frac{4x-20+23}{x-5} dx = \int \frac{4x-20}{x-5} + \frac{23}{x-5} dx = \int \frac{4(x-5)}{x-5} + \frac{23}{x-5} dx = \int 4 + \frac{23}{x-5} dx =$$

$$= \int 4 dx + \int \frac{23}{x-5} dx = [*] 4x + \int \frac{23}{t} dt = 4x + 23 \ln |t| + c = 4x + 23 \ln |x - 5| + c$$

$$* \quad t = x - 5$$

$$\frac{dt}{dx} = 1$$

$$dt = dx$$

$$\int \frac{5x+7}{2x-1} dx = \int \frac{5x-2,5+9,5}{2x-1} dx = \int \frac{5x-2,5}{2x-1} + \frac{9,5}{2x-1} dx = \int \frac{2,5(2x-1)}{2x-1} + \frac{9,5}{2x-1} dx = \int \frac{5}{2} + \frac{19}{2(2x-1)} dx =$$

$$= \int \frac{5}{2} dx + \frac{19}{2} \int \frac{1}{2x-1} dx = [*] \frac{5}{2} \cdot x + \frac{19}{2} \int \frac{1}{t} \frac{dt}{2} = \frac{5x}{2} + \frac{19}{4} \ln |t| + c = \frac{5x}{2} + \frac{19}{4} \ln |2x-1| + c$$

$$* \quad t = 2x - 1$$

$$\frac{dt}{dx} = 2$$

$$\frac{dt}{2} = dx$$

$$\int \frac{4x^2-6x-5}{x+3} dx = \int \frac{4x^2-6x-54+49}{x+3} dx = \int \frac{4x^2-6x-54}{x+3} + \frac{49}{x+3} dx = \int \frac{(x+3)(4x-18)}{x+3} + \frac{49}{x+3} dx =$$

$$= \int 4x - 18 + \frac{49}{x+3} dx = \int 4x - 18 dx + \int \frac{49}{x+3} dx = [*] 4 \cdot \frac{x^2}{2} - 18 \cdot x + \int \frac{49}{t} dt = 2x^2 - 18x + 49 \ln |t| + c =$$

$$= 2x^2 - 18x + 49 \ln |x+3| + c$$

elosztjuk: $(4x^2 - 6x - 5) : (x + 3) = 4x - 18$ a maradék pedig +49

$$* \quad t = x + 3$$

$$\frac{dt}{dx} = 1$$

$$dt = dx$$

$$\int \frac{3x+1}{2x^2-5x-3} dx =$$

$$\frac{3x+1}{2x^2-5x-3} = \frac{3x+1}{(2x+1)(x-3)} = \frac{A}{2x+1} + \frac{B}{x-3} \quad \text{kde } A, B \in \mathbb{R}$$

$$\frac{A}{2x+1} + \frac{B}{x-3} = \frac{A(x-3)+B(2x+1)}{(2x+1)(x-3)} = \frac{Ax-3A+2Bx+B}{(2x+1)(x-3)} = \frac{x(A+2B)+(-3A+B)}{(2x+1)(x-3)}$$

$$\frac{x(A+2B)+(-3A+B)}{(2x+1)(x-3)} = \frac{3x+1}{(2x+1)(x-3)}$$

ebből egyenletrendszert kapunk:

$$(A + 2B) \cdot x = 3x \rightarrow A + 2B = 3$$

$$\frac{-3A + B = 1}{A + 2B = 3} \quad / \cdot (-2)$$

$$A + 2B = 3$$

$$\frac{6A - 2B = -2}{7A = 1} \quad / I. + II.$$

$$7A = 1$$

$$A = \frac{1}{7}$$

$$-3 \cdot \frac{1}{7} + B = 1$$

$$B = \frac{10}{7}$$

$$\int \frac{3x+1}{2x^2-5x-3} dx = \int \frac{\frac{1}{7}}{2x+1} + \frac{\frac{10}{7}}{x-3} dx = \frac{1}{7} \int \frac{1}{2x+1} dx + \frac{10}{7} \int \frac{1}{x-3} dx = [*] \frac{1}{7} \int \frac{1}{t} \frac{dt}{2} + [\nabla] \frac{10}{7} \int \frac{1}{u} du =$$

$$= \frac{1}{14} \ln |t| + \frac{10}{7} \ln |u| + c = \frac{1}{14} \ln |2x+1| + \frac{10}{7} \ln |x-3| + c$$

$$* \quad t = 2x + 1$$

$$\frac{dt}{dx} = 2$$

$$\frac{dt}{2} = dx$$

$$\nabla \quad u = x - 3$$

$$\frac{du}{dx} = 1$$

$$du = dx$$